XII. *Radiation in the Solar System: its Effect on Temperature and its Pressure on Small Bodies.*

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PART I.

TEMPERATURE.

WHEN a surface is a full radiator and absorber* its temperature can be determined at once by the fourth-power law if we know the rate at which it is radiating energy. If it is radiating what it receives from the sun, then a knowledge of the solar constant enables us to find the temperature. We can thus make estimates of the highest temperature which a surface can reach when it is only receiving heat from the sun. We can also make more or less approximate estimates of the temperatures of the planetary surfaces by assuming conditions under which the radiation takes place,† and we can determine, fairly exactly, the temperatures of very small bodies in interplanetary space.

These determinations require a knowledge of the constant of radiation and of either the solar constant or the effective temperature of the sun, either of which, as is well known, can be found from the other by means of the radiation constant. It will be convenient to give here the values of these quantities before proceeding to apply them to our special problems.

* A surface which absorbs, and therefore emits, every kind of radiation, is usually described as "black," a description which is obviously bad when the surface is luminous. It is much better described as "a full absorber" or "a full radiator."

† This was pointed out by W. WIEN in his report on "Les Lois Théoriques du Rayonnement" ('Congrès International de Physique,' vol. ii., p. 30). He remarks that STEFAN'S law enables us to calculate the temperatures of celestial bodies which receive their light from the sun, by equating the energy which they radiate to the energy which they receive from the sun, and states that for the earth we obtain nearly the mean temperature, using the reflecting power of Mars, while the temperature of Neptune should be below $-200°$ C.

The Constant of Radiation.

If R is the energy radiated per second per square centimetre by a full radiator at temperature $\theta°$ A (where A stands for the absolute scale), the fourth-power law states that

$$R = \sigma\theta^4,$$

where σ is the constant of radiation.

According to KURLBAUM* the constant is

$$\sigma = 5 \cdot 32 \times 10^{-5} \text{ erg}.$$

The Solar Constant.

The solar constant is usually expressed as a number of calories received per minute by a square centimetre held normal to the sun's rays at the distance of the earth. The determinations by different observers differ so widely that it is not necessary for our present purpose to consider whether the constant really exists or whether there are small periodic variations from constancy.

ÅNGSTRÖM estimated the value as 4 calories per square centimetre per minute, and this value is adopted by CROVA as very probable.† When converted to ergs per second this gives

$$S_a = 0 \cdot 28 \times 10^7 \text{ ergs/cm.}^2 \text{ sec.},$$

where the suffix denotes that it is ANGSTRÖM's value.

LANGLEY‡ assumed that the atmosphere transmits about 59 per cent. of the energy from a zenith sun and from his measurement of the heat reaching the earth's surface he estimated the value of the constant at 3 calories. This gives

$$S_l = 0 \cdot 21 \times 10^7 \text{ ergs/cm.}^2 \text{ sec.},$$

ROSETTI§ assumed a transmission of 78 per cent. from the zenith sun, but WILSON and GRAY|| consider that 71 per cent. represents ROSETTI's numbers better than 78 per cent. If in LANGLEY's value we replace 59 per cent. by 71 per cent. we get 2·5 calories. This gives

$$S_r = 0 \cdot 175 \times 10^7 \text{ ergs/cm.}^2 \text{ sec.}$$

* 'Wied. Ann.,' vol. 65, 1898, p. 748.

† 'Congrès International de Physique,' vol. 3, p. 453.

‡ 'Phil. Mag.,' vol. 15, 1883, p. 153, and 'Researches on Solar Heat.'

§ 'Phil. Mag.,' vol. 8, 1879, p. 547.

|| 'Phil. Trans.,' A, 1894, p. 383.

The Radiation from the Sun's Surface.

If s is the radius of the sun's surface, R the radiation per square centimetre, then the total rate of emission is $4\pi s^2 R$. This passing through the sphere of radius r, at the distance of the earth and with surface $4\pi r^2$, gives

$$4\pi s^2 R = 4\pi r^2 S,$$

where S is the solar constant.

Hence

$$R = \frac{r^2}{s^2} S = \left(\frac{9 \cdot 23 \times 10^7}{4 \cdot 3 \times 10^5}\right)^2 S = 46{,}000 S.$$

Corresponding to the three values of S just given we have three values of R, viz.,

$$R_a = 1 \cdot 29 \times 10^{11}; \qquad R_l = 0 \cdot 945 \times 10^{11}; \qquad R_r = 0 \cdot 805 \times 10^{11}.$$

The Effective Temperature of the Sun.

If we equate the sun's radiation to $\sigma\theta^4$, where σ is the radiation constant, we get θ, the "effective temperature" of the sun, that is the temperature of a full radiator which is emitting energy at the same rate.

Thus

$$5 \cdot 32 \times 10^{-5}\, \theta_a{}^4 = 1 \cdot 29 \times 10^{11},$$

whence

$$\theta_a = 7000^\circ \text{ A approximately.}$$

Similarly

$$\theta_l = 6500^\circ \text{ A}; \qquad \theta_r = 6200^\circ \text{ A}.$$

Wilson* made a direct comparison of the radiation from the sun with that from a full radiator at known temperature. Assuming a zenith transmission of 71 per cent., he obtained 5773° A as the effective solar temperature. If we put

$$46{,}000 S = 5 \cdot 32 \times 10^{-5} \times 5773^4,$$

we get

$$S = 0 \cdot 128 \times 10^7.$$

This is no doubt too low a value. Either then Wilson's zenith transmission was less than 71 per cent. or Kurlbaum's constant is too small.

The low value is probably to be accounted for chiefly by the first supposition. Wilson points out that if x is the true value of the transmission, his value of the temperature is to be multiplied by $(71/x)^{\frac{1}{4}}$. If we take $\theta_r = 6200^\circ$ as the true value then x will be given by

$$x = \left(\tfrac{5773}{6200}\right)^4 \times 71 = 53.$$

This low value is not necessarily inconsistent with the much higher value 71 per cent.

* 'Roy. Soc. Proc.,' vol. 69, 1901–2, p. 312.

used above in finding ROSETTI's solar constant, for no doubt the transmission varies widely with time and place, and we have no reason to assume that 1·77 calories per minute, obtained by LANGLEY, would have been received from the zenith at the time and in the place where WILSON was making his determination.

The Effective Temperature of Space.

In determining the steady temperature of any body as conditioned by the radiation received from the sun, we have to consider whether it is necessary to take into account the radiation from the rest of the sky. If it receives S from the sun, ρ from the rest of the sky, and if its own radiation is R, then in the steady state

$$R = S + \rho \quad \text{or} \quad R - \rho = S.$$

It behaves therefore as if it were receiving S from the sun, but as if it were placed in a fully radiating enclosure of such temperature that the radiation is ρ. This temperature is the "effective temperature of space."

The temperature may perhaps be more definitely described as that of a small full absorber placed at a distance from any planet and screened from the sun. Various well-known attempts have been made to estimate this temperature, but the data are very uncertain. The fourth-power law however shows that it is not very much above the absolute zero, if we can assume that the quality of starlight is not very different from that of sunlight.

According to L'HERMITE * starlight is one-tenth full moonlight. Full moonlight is variously estimated in terms of full sunlight. LANGLEY † takes it as $\frac{1}{400,000}$. These two values combined give sunlight as 4×10^6 starlight. But starlight comes from the whole hemisphere, while the sun only occupies a small part of it. In comparing temperatures we have to use the brightness of sunlight as if the whole hemisphere were paved with suns.

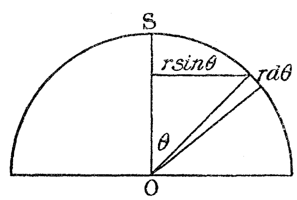

Fig. 1.

If B is the illumination of a surface at O, fig. 1, lighted by the sun in the zenith at S, and if πs^2 is the area of the sun's diametral plane, then $B/\pi s^2$ is the illumination at O due to each square centimetre. If the hemisphere were all of the same brightness as the sun, the illumination at O due to the ring of sky between θ and $\theta + d\theta$ would be

$$\frac{B}{\pi s^2} 2\pi r^2 \sin\theta \cos\theta \, d\theta,$$

where r is the distance of the sun.

* 'L'Astronomie,' vol. 5, p. 406.

† "First Memoir on the Temperature of the Surface of the Moon." 'National Academy of Sciences vol. 3.

Integrating from $\theta = 0$ to $\theta = \pi/2$, we have

$$\text{Total illumination} = \mathrm{B}r^2/s^2 = 46{,}000\ \mathrm{B}.$$

The illumination from a hemisphere paved with suns is therefore $46{,}000 \times 4 \times 10^6 = 1{\cdot}84 \times 10^{11}$ times that from the stellar sky.

If we assume that the quality of the radiation is the same in both cases, that is, if we assume that the energy is proportional to the light part of the spectrum, we have by the fourth-power law

$$\text{Effective temperature of space} = \frac{\text{effective temperature of sun}}{({\cdot}184 \times 10^{12})^{\frac{1}{4}}}$$

$$= \frac{\text{effective temperature of sun}}{655}.$$

As the temperature of the sun probably lies between 6000° A and 7000° A, this gives

$$\text{Effective temperature of space} = 10^\circ\ \mathrm{A}.$$

If, then, a body is raised by the sun to even such a small multiple of 10° as, say, 60°, the fourth-power law of radiation implies that it is giving out and therefore receiving from the sun more than a thousand times as much energy as it is receiving from the sky.

The sky radiation may therefore be left out of the account when we are dealing with approximate estimates and not with exact results, and bodies in the solar system may be regarded as being situated in a zero enclosure except in so far as they receive radiation from the sun.

Temperature of a Planet under Certain Assumed Conditions when placed at a distance from the Sun equal to that of the Earth.

The real earth presents a problem of complexity far too great to deal with. I shall therefore consider an ideal earth for which certain conditions hold, more or less approximating to reality, and determine the temperature of its surface on the assumption that it receives heat from the sun only.

Let us suppose :—

1. That the planet is rotating about an axis perpendicular to the plane of its orbit, which is circular.

This will give us too high a temperature at the equator, and the absolute zero, which is too low, at the poles. The mean, however, over the planet, will probably be not much affected by the supposition.

2. That the effect of the atmosphere is to keep the temperature in any given latitude the same, day and night.

This is not a great departure from reality. On the sea, which is more than twothirds of the earth's surface, the daily range is very small, of the order of 1° or 2° C., while even on the land it is, in extreme cases, not more than 15° C., which is not a large fraction of the absolute temperature.

3. That the surface and the atmosphere over it at any one point have one effective temperature as a full radiator. This is no doubt a departure from reality. How wide a departure we have no present means of estimating.

4. That there is no convection of heat from one latitude to another.

This is a very wide departure from reality. But, as we shall see below, the mean temperature of the planet is very little affected by convection, even if we assume that it is so extensive as to make the surface of uniform temperature.

5. That the reflexion at each point is $\frac{1}{10}$th of the radiation received.

This is probably of the order of the actual reflexion from the earth. According to LANGLEY* the moon reflects about $\frac{1}{8}$th of the radiation received. The earth certainly reflects less. The temperatures determined hereafter are proportional to the 4th root of the coefficient of absorption. Even if this coefficient is as low as 0·9 its 4th root is 0·974. Hence if the actual value is anywhere between 0·9 and 1, the assumed value of 0·9 will not make an error of more than $2\frac{1}{2}$ per cent. in the value of the temperature.

6. That the planet ultimately radiates out all the heat received from the sun, no more and no less.

This again is very near the condition of the real earth, which, on the whole, radiates out rather more than it receives—perhaps on the average a calorie per square centimetre in three days.

Making these six suppositions, let us calculate the temperature of various parts of this ideal planet.

Consider a band between latitudes λ and $\lambda + d\lambda$. The area receiving heat from the sun at any instant, if projected normally to the stream of solar radiation, is (fig. 2)

$$2r \cos \lambda \ r d\lambda \cos \lambda = 2r^2 \cos^2 \lambda d\lambda,$$

where r is the radius of the planet.

If S is the solar constant, this band is absorbing, with coefficient 0·9,

$$0{\cdot}9 S \times 2r^2 \cos^2 \lambda d\lambda.$$

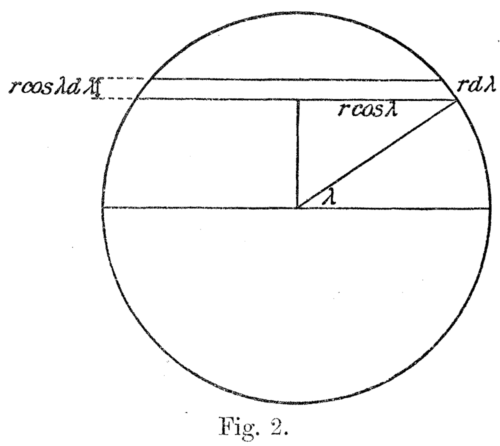

Fig. 2.

But the band all round the globe is radiating equally, according to the second supposition, and the radiating area is

$$2\pi r \cos \lambda \, . \, r d\lambda = 2\pi r^2 \cos \lambda d\lambda.$$

* "Third Memoir on the Temperature of the Moon." 'National Academy of Sciences,' vol. 4, Part 2, p. 197.

Hence the radiation emitted per square centimetre is

$$\frac{0{\cdot}9\ \mathrm{S}\ 2r^2 \cos^2\lambda\, d\lambda}{2\pi r^2 \cos\lambda\, d\lambda} = \frac{0{\cdot}9\ \mathrm{S} \cos\lambda}{\pi}.$$

If the effective temperature in this latitude is θ_λ, we have

$$\frac{0{\cdot}9\ \mathrm{S} \cos\lambda}{\pi} = 5{\cdot}32 \times 10^{-5}\theta_\lambda{}^4,$$

or

$$\theta_\lambda = \left(\frac{0{\cdot}9 \times 10^5 \mathrm{S}}{5{\cdot}32\pi}\right)^{\frac{1}{4}} \cos^{\frac{1}{4}}\lambda.$$

If we put $\lambda = 0$, we get the equatorial temperature corresponding to each of the different values of S given above, viz. :

Equatorial $\theta_a = 350°$ A approximately.
,, $\theta_l = 325°$ A ,,
,, $\theta_r = 312°$ A ,,

The temperature in latitude λ is

$$\theta_\lambda = \text{equatorial temperature} \times \cos^{\frac{1}{4}}\lambda.$$

Thus, in latitude 45°, it is 0·917 equatorial temperature.

The average temperature over the globe is

$$\frac{2}{4\pi r^2}\int_0^{\frac{\pi}{2}} 2\pi r^2 \cos\lambda\theta_{\mathrm{E}} \cos^{\frac{1}{4}}\lambda\, d\lambda,$$

where θ_{E} is the equatorial temperature

$$= \theta_{\mathrm{E}}\int_0^{\frac{\pi}{2}} \cos^{\frac{5}{4}}\lambda\, d\lambda = \theta_{\mathrm{E}}\frac{\sqrt{\pi}}{2}\,\frac{\Gamma\left(\frac{9}{8}\right)}{\Gamma\left(\frac{5}{8}\right)} = 0{\cdot}93\theta_{\mathrm{E}}.$$

The average temperature, then, is little more than 1 per cent. above the temperature in latitude 45°.

If we use the three values of θ_{E} just given, we have

Average $\theta_a = 325°$ A approximately.
,, $\theta_l = 302°$ A ,,
,, $\theta_r = 290°$ A ,,.

Our fourth supposition was that there is no convection by wind or water from one latitude to another. Let us now go to the other extreme and suppose that the convection is so great that the temperature is practically uniform all over the globe.

We then have a receiving surface virtually πr^2, and a radiating surface $4\pi r^2$. Then we get the radiation emitted per square centimetre

$$\frac{0 \cdot 9 S \pi r^2}{4\pi r^2} = \frac{9S}{40};$$

and if θ is the temperature required for this,

$$5 \cdot 32 \times 10^{-5} \theta^4 = \frac{9S}{40};$$

whence

$$\text{Uniform } \theta_a = 330^\circ \text{ A approximately}$$
$$\text{,,} \quad \theta_l = 307^\circ \text{ A} \quad \text{,,}$$
$$\text{,,} \quad \theta_r = 293^\circ \text{ A} \quad \text{,,}$$

values not more than 5° above those obtained for the average on the supposition of no convection.

Comparing these results with the temperature of the real earth, it is seen at once that they are of the same order.

The average temperature of the earth's surface is usually estimated at about 60° F., say 289° A. The temperature of the atmosphere is on the whole decidedly lower than that of the surface below it. We should therefore conclude that the earth's effective temperature is somewhat below 289° A.

Again, the earth and the atmosphere, taken as one surface, do not constitute a full absorber, but are to some extent selective. Hence we should expect the earth to be, if anything, of a higher temperature than a full absorber and radiator under the same conditions.

For both these reasons, then, the ideal planet might be expected to have a temperature below rather than above 289° A. The lowest estimate obtained above is therefore probably nearest to the truth, and it would appear that even that is somewhat too high. This tends to show that, if we accept KURLBAUM'S value of the radiation constant, we cannot put the solar constant so high as 3 or 4, but must accept a value much nearer to that which I have called ROSETTI'S value, viz., 2·5.

In what follows I shall therefore take ROSETTI'S value and the resulting value of the solar temperature, viz., 6200° A.

The calculation made above may be turned the other way round, and may be used for a

Determination of the Effective Temperature of the Sun from the Average Temperature of the Earth.

Assuming that the real earth may be replaced by the ideal planet already considered, the radiation per square centimetre from the equatorial band is $\frac{0 \cdot 9S}{\pi}$. But the radiation per square centimetre from the sun's surface is 46,000S. If then θ_E is the earth's equatorial temperature, and θ_S is the solar temperature,

$$\frac{0 \cdot 9S}{\pi} : 46,000S = \theta_E^4 : \theta_S^4,$$

whence

$$\theta_E = \theta_S/20.$$

The average temperature of the earth is 0·93 of the equatorial temperature. If this average is θ_A, then

$$\theta_A = \theta_S/21 \cdot 5.$$

If we take the temperature of the real earth as 289° A, and as being equal to that of the ideal,

$$\theta_S = 21 \cdot 5 \times 289° = 6200° \text{ A approximately.}$$

Upper Limit to the Temperature of a Fully Radiating Surface exposed normally to Solar Radiation at the Distance of the Earth from the Sun.

The highest temperature which a full radiator can attain is that for which its radiation is equal to the energy received. This will only hold when no appreciable quantity of heat is conducted inwards from the surface.

To obtain the upper limit in the case under consideration, we have to equate the radiation to the solar constant, which we shall now take as $S_r = 0 \cdot 175 \times 10^7$. Then,

$$5 \cdot 32 \times 10^{-5} \theta^4 = 0 \cdot 175 \times 10^7,$$

and

$$\theta = 426° \text{ A}.$$

If the surface reflects some of the radiation and absorbs a fraction x of that falling on it, then the effective temperature is

$$x^{\frac{1}{4}} \times 426° \text{ A}.$$

The Limiting Temperature of the Surface of the Moon.

We may apply this result to find an upper limit to the temperature of the moon's surface. This upper limit can only be attained when it is sending out radiation as rapidly as it receives it, and is therefore conducting no appreciable quantity inwards.

We shall take LANGLEY's estimate (*loc. cit.*) of $\frac{\text{reflected radiation}}{\text{emitted radiation}} = \frac{1}{6 \cdot 7}$. This is represented nearly enough by $x = \frac{7}{8}$.

The upper limit of temperature of the surface exposed to a zenith sun is, therefore,

$$\theta = 426 \times (\tfrac{7}{8})^{\frac{1}{4}} = 426 \times 0 \cdot 967 = 412° \text{ A}.$$

This, then, is the upper limit to the temperature of the hottest part of an airless moon.

For a surface at angle λ with the line to the sun,

$$\theta_\lambda = 412 \cos^{\frac{1}{4}} \lambda.$$

If we take this as the law of temperature of the side of the moon exposed to the sun, we can find the effective temperature of the full moon as seen from the earth, *i.e.*, the uniform temperature of a flat disc of radius equal to that of the moon, sending to us the same total radiation.

If $N d\omega$ is the normal stream of radiation from 1 sq. centim. of surface of the moon immediately under the sun sent out through a cone angle $d\omega$, that sent out in direction λ to the normal is $N \cos \lambda \, d\omega$. But 1 sq. centim. on the moon's surface inclined at λ to the sun's rays only receives cos λ of the radiation received by the surface immediately under the sun. It therefore sends in the direction of the earth, also at λ to the normal, only $N \cos^2 \lambda \, d\omega$. Hence the total radiation to the earth, obtained by putting $d\omega = 2\pi \sin \theta \, d\theta$ and integrating is

$$\int_0^{\frac{\pi}{2}} \frac{N \cos^2 \lambda \; 2\pi m^2 \sin \lambda \, d\lambda}{r^2},$$

where m is the radius of the moon and r is its distance from the earth

$$= \frac{2\pi}{3} \frac{m^2}{r^2} N.$$

Let N_D be the normal stream from the equivalent flat disc, then

$$\frac{\pi m^2 N_D}{r^2} = \tfrac{2}{3}\pi \frac{m^2}{r^2} N$$

and

$$N_D = \tfrac{2}{3} N.$$

The effective temperature of the flat disc is therefore $\sqrt[4]{\frac{2}{3}}$ that of the surface immediately under the sun at the same distance from it.

Then the effective average $= 412 \times \sqrt[4]{\frac{2}{3}} = 412 \times 0{\cdot}9 = 371°$ A. The upper limit, then, to the average effective temperature of the moon's disc is just below that of boiling water.

This is very considerably above LANGLEY'S estimate, that the surface of the full moon is a few degrees above the freezing-point. There can be no doubt that a very appreciable amount of heat is conducted inwards. The observations during eclipses by LANGLEY* and by BOEDDICKER show that some heat is still received from the moon's surface when it has entered the full shadow, and that it takes time after the eclipse has passed to establish a steady temperature again. It might be possible to

* 'Third Memoir,' p. 159.

make some rough estimate of the amount conducted inwards from the Fourier equation, but the problem is not an easy one. Perhaps we get the best estimate by comparing the actual temperature with that above found.

If the actual temperature is taken as about $\frac{4}{5}$ the upper limit, say 297° A, then the radiation outwards is of the order $\sqrt[4]{\frac{4}{5}} = 0{\cdot}41$ of that where no conduction exists. Then nearly $\frac{3}{5}$ of the heat is probably conducted inwards.

If the moon always turned the same face to the sun instead of to the earth, the upper limit would be approached.

Temperature of a Spherical Absorbing Solid Body of the Order 1 *centim. in diameter at the Distance of the Earth from the Sun.*

The calculation of the temperature of such a body is interesting for two reasons. Firstly, the body will be at nearly the same temperature throughout, and secondly, as we shall show in the second part of this paper, the mutual repulsion of two such bodies, due to the pressure of their radiation, is of the same order as their gravitative attraction.

If the radius of the body is a, its effective receiving area is πa^2, and it receives

$$\pi a^2 \text{S ergs/sec.}$$

Its radiating surface is $4\pi a^2$, and therefore its average radiation per square centimetre in the steady state is

$$\pi a^2 \text{S}/4\pi a^2 = \tfrac{1}{4}\text{S}.$$

If we take $\text{S} = 2{\cdot}5$ cal./min. or $0{\cdot}04$ cal./sec., and if the conductivity is of the order of that of terrestrial rock lying, say, between $0{\cdot}01$ and $0{\cdot}001$, it is evident that a difference of temperature of only a few degrees between the receiving and the dark surfaces will convey heat sufficient to supply radiation, $0{\cdot}01$ cal./sec., equal to the average. Thus, if the conductivity is $0{\cdot}001$ and the diameter is 1 centim., a difference of temperature of 10° suffices.

We may therefore take the temperature of the surface as approximately uniform when the steady state is reached. Let the temperature be θ, and let the solar temperature be θ_{S}. Then we have

$$\theta^4 : \theta_{\text{S}}^4 = \frac{\text{S}}{4} : 46{,}000\ \text{S}$$

and

$$\theta = \frac{\theta_{\text{S}}}{20{\cdot}7}.$$

If

$$\theta_{\text{S}} = 6200°\ \text{A},$$

$$\theta = 300°\ \text{A approximately.}$$

This will be the temperature of fully absorbing bodies smaller than 1 centim., so long as they are not too small to absorb the radiation falling on them.

Variation of Temperature with Distance from the Sun.

Since the radiation received varies inversely as the square of the distance from the sun, that given out varies in the same ratio. The temperature of the radiating surface varies therefore as the fourth root of the inverse square, that is inversely as the square root of the distance.

This enables us to deduce at once the temperatures of the various surfaces and bodies which we have considered, if placed at the distances of different planets as well as at the distance of the earth. We have merely to multiply the results hitherto found by $\sqrt{\frac{\text{Earth's distance}}{\text{Planet's distance}}}$.

The following table contains the values of the temperatures at selected distances, all on the absolute scale :—

TABLE of Temperatures of Surfaces at Different Distances from the Sun. All on the Absolute Scale.

At the distance of the planet. I.	Distance. Earth's distance = 1. II.	Square root of (distance)$^{-1}$. III.	Equatorial temperature of ideal planet. IV.	Average temperature of ideal planet. V.	Upper limit of a surface reflecting one-eighth under zenith sun. VI.	Average temperature of equivalent disc. VII.	Temperature four-fifths that of equivalent disc. VIII.	Temperature of small absorbing sphere. IX.
Mercury .	0·3871	1·61	502	467	664	598	478	483
Venus .	0·7233	1·18	368	342	486	438	350	358
Earth . .	1·0000	1·00	312	290	412	371	297	300
Mars . .	1·5237	0·81	253	235	337	300	240	243
Neptune .	30·0544	0·18	56	52	74	67	53	54

We have omitted the larger planets except Neptune, since in all probability they radiate heat of their own in considerable proportion. Neptune is inserted merely to show how low temperatures would be at his distance if there were no supply of internal heat.

The results given in the table may not be exactly applicable to any of the planets, but they at least indicate the order of temperature which probably prevails.

If, for instance, Mars is to be regarded as having an atmosphere with regulating properties like our own, his equatorial temperature, Column IV., is probably far below the temperature of freezing water, and his average temperature, Column V., must be not very different from that of freezing mercury. If, on the other hand, we suppose that his atmosphere has no regulating power, we get the upper limits not very different from those in Columns VI. and VII. These are the limits for the bright side, and they imply nearly absolute zero on the dark side. If we regard Mars as resembling our moon, and take the moon's effective average temperature as 297° A, the corresponding temperature for Mars is 240° A, and the highest temperature is $\frac{4}{5} \times 337 = 270°$. But the surface of Mars has probably a higher coefficient of absorption than the surface of the moon—it certainly has for light—so that we may put his effective average temperature on this supposition some few degrees above 240° A, and his equatorial temperature some degrees higher still.

It appears exceedingly probable, then, that whether we regard Mars as like the earth, or, going to the other extreme, as like the moon, the temperature of his surface is everywhere below the freezing-point of water. The only escape from this conclusion that I can see is by way of a supposition that an appreciable amount of heat is issuing from beneath his surface.

We cannot draw any definite conclusions as to the temperatures of Mercury and Venus till we know whether they have atmospheres and whether they rotate on their own axes. If we make both these suppositions and further suppose that their conditions approximate to those of the ideal planet at their distances given in Columns IV. and V., then they may well be surrounded by hot clouds, as is sometimes supposed, entirely screening their solid bodies from us. If, on the other hand, their atmospheres are ineffective as regulators and if they always present the same face to the sun, the hottest part of Mercury is probably not far from 650° A, and that of Venus not far from 500° A.

If a comet consist of small solid particles of diameter of the order 1 centim. or less, then the temperatures of these particles are given in Column IX. At one-quarter of the earth's distance, say 23 million miles from the sun, the temperature is 600°, about the melting-point of lead. At one-twenty-fifth, say $3\frac{3}{4}$ million miles, it will be about 1500°, say the melting point of cast-iron. Nearer than this the temperature no doubt increases rapidly, but the law of temperature, deduced from the inverse square law for the radiation received, requires amendment, as that law was based on the supposition that a hemisphere only is lighted by the sun, and that the whole of his disc is visible from every part of that hemisphere. Both of these suppositions cease to hold when the distance from the sun is only a small multiple of his radius.

PART II.

Radiation Pressures.

The pressure of radiation against a surface on which it falls, first deduced by Maxwell from the Electromagnetic Theory of Light, is now established on an experimental basis by the work of Lebedew, confirmed by that of Nichols and Hull.

Though this pressure was first deduced as a consequence of the Electromagnetic Theory, BARTOLI showed, independently, that a pressure must exist without any theory as to the nature of light beyond a supposition which may perhaps be put in the form that a surface can move through the ether, doing work on the radiation alone and not on the ether in which the radiation exists. Professor LARMOR* has given a proof of this pressure and has shown that it has the value assigned to it by MAXWELL, viz., that it is numerically equal to the energy density in the incident wave, whatever may be the nature of the waves, so long as their energy density for given amplitude is inversely as the square of the wave-length. We may, in fact, regard a pencil of radiation as a stream of momentum, the direction of the momentum being the axis of the pencil. If E is the energy density of the pencil, U its velocity, the momentum density may be regarded as E/U.

If the stream of radiation is being emitted by a surface, the surface is losing the momentum carried out with the issuing stream, and is so being pressed backwards. If the stream is being absorbed by the surface, then it is gaining the momentum and is still being pressed backwards, the forces being in the line of propagation.

As the expressions for the radiation pressure in various cases are probably not very well known, it may be convenient to state them here for use in what follows.

Values of Radiation Pressure in Different Cases.

If 1 sq. centim. of a full radiator is emitting energy R per second, and if $N d\omega$ is the energy it is emitting through a cone $d\omega$, with axis along the normal, then in direction θ its projection is $\cos\theta$, and it is emitting $N \cos\theta d\omega$ through a cone $d\omega$. Putting $d\omega = 2\pi \sin\theta\, d\theta$, and integrating over the hemisphere, we have

$$R = \int_0^{\frac{\pi}{2}} N \cos\theta \,.\, 2\pi \sin\theta\, d\theta = \pi N.$$

If we draw a hemisphere, radius r, round the source as centre, the energy falling on area $r^2 d\omega$ is $N \cos\theta\, d\omega$ per second, and, since the velocity is U per second, the energy density just outside the surface on which it falls is $N \cos\theta/Ur^2$, and this is the rate at which the momentum is being received, that is, it is the normal pressure. The total force on area $r^2 d\omega$ is $N \cos\theta\, d\omega/U$. This is the momentum sent out by the radiating square centimetre per second through the pencil with angle $d\omega$, in the direction θ, and is therefore the force on the square centimetre due to that pencil.

Resolving along the normal and in the surface we have

$$\text{Normal pressure} = N \cos^2\theta\, d\omega/U.$$

$$\text{Tangential stress} = N \cos\theta \sin\theta\, d\omega/U.$$

* 'Brit. Assoc. Report,' 1900; 'Encyc. Brit.,' vol. 32, Art. "Radiation."

Putting $d\omega = 2\pi \sin \theta \, d\theta$ and integrating over the hemisphere, we get

$$\text{Total normal pressure} = \int_0^{\frac{\pi}{2}} (\text{N} \cos^2 \theta \,.\, 2\pi \sin \theta d\theta/\text{U}) = 2\pi\text{N}/3\text{U} = 2\text{R}/3\text{U}.$$

Total tangential stress $= 0$, since the radiation is symmetrical about the normal.

If the surface is receiving radiation, let us suppose that the stream is a parallel pencil S per second per square centimetre held normal to the stream, and that it is inclined at θ to the normal to the receiving surface. The momentum received per second is $\text{S} \cos \theta/\text{U}$. This produces

$$\text{Normal pressure} = \text{S} \cos^2 \theta/\text{U}.$$
$$\text{Tangential stress} = \text{S} \cos \theta \sin \theta/\text{U}.$$

If the stream is entirely absorbed both these forces exist.

If the stream is entirely reflected, the reflected pencil exerts an equal normal force and an equal and opposite tangential force, and we have only normal pressure of amount $2\text{S} \cos^2 \theta/\text{U}$.

If only a fraction μ is reflected, the incident and reflected streams will give

$$\text{Normal pressure} = (1 + \mu)\,\text{S} \cos^2 \theta/\text{U}.$$
$$\text{Tangential stress} = (1 - \mu)\,\text{S} \cos \theta \sin \theta/\text{U}.$$

To the normal pressure must be added the pressure due to the radiation emitted from the surface.

Radiation Pressure in Full Sunlight.

If a full absorber is exposed normally to the solar radiation at the distance of the earth the pressure on it is S/U, or $\dfrac{0{\cdot}175 \times 10^7}{3 \times 10^{10}} = 5{\cdot}8 \times 10^{-5}$ dyne/sq. centim.

The Radiation Pressures Between Small Bodies. Comparison with their mutual Gravitation.

It is well known that the radiation force on a small body, exposed to solar radiation, does not decrease so rapidly as gravitative pull on the body as its size decreases. If the body is a sphere of radius a and density ρ, and with a fully absorbing surface, and if it is so small that it is practically at one temperature all through, it is receiving a stream of momentum

$$\pi a^2 \text{S}/\text{U}$$

directed from the sun. Its own radiation outwards being equal in all directions has zero resultant pressure.

The gravitative acceleration towards the sun at the distance of the earth is about 0·59 centim./sec.^{2}. Then we have

$$\frac{\text{Radiation pressure}}{\text{Gravitation pull}} = \frac{\pi a^2 \mathrm{S}}{\mathrm{U} \times \frac{4}{3}\pi a^3 \rho \times 0{\cdot}59}.$$

The two will be equal when

$$a = \tfrac{3}{4}\,\frac{\mathrm{S}}{\mathrm{U}\rho \times 0{\cdot}59}.$$

If we put

$$\rho = 1\,;\ \mathrm{S} = 0{\cdot}175 \times 10^7\,;\ \mathrm{U} = 3 \times 10^{10}\,;$$

we get

$$a = 74 \times 10^{-6}.$$

This is the well-known result that a body of diameter about two wave-lengths of red light would be equally attracted and repelled if we could assume that a surface so small still continued to absorb. But, of course, when we are getting to dimensions comparable with a wave-length that assumption can no longer be made.

It is not, I think, equally well recognised that if the radiating body is diminished in size, the radiation pressure due to it also decreases less rapidly than the gravitative pull which it exerts. For the radiation decreases as the square of the radius of the emitting body and its gravitative pull as the cube.

We can easily compare the radiation and gravitation forces between two bodies, if for simplicity we assume that their distance apart is very great compared with the radius of either.

Let AB, fig. 3, be two spheres with full radiating surfaces. Let their radii be

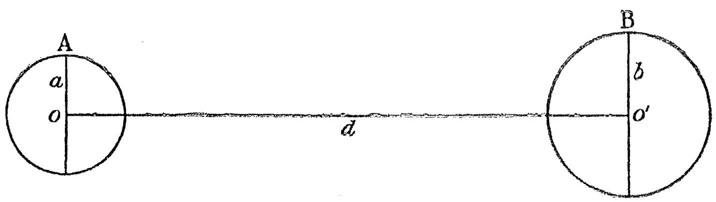

Fig. 3.

a, b and let their centres oo' be d apart. If this distance is great compared with a and b, each may be regarded as receiving a parallel stream from the other.

Let A send out a normal stream $\mathrm{N}\,d\omega$ per square centimetre through cone $d\omega$, while B sends out $\mathrm{N}'\,d\omega$.

B receives the stream of cross section πb^2 or the angle of the cone is $\pi b^2/d^2$, and it issues virtually from area πa^2, for at B, A will appear as a uniformly bright flat disc.

Then the total force on B is

$$\frac{\pi a^2 \mathrm{N}}{\mathrm{U}} \times \frac{\pi b^2}{d^2} = \frac{\pi a^2 b^2 \mathrm{R}}{\mathrm{U} d^2},$$

where

$$\mathrm{R} = \pi \mathrm{N}.$$

The force on A due to B is $\pi a^2 b^2 R'/Ud^2$, where $R' = \pi N'$.

These are not equal unless $R = R'$, *i.e.*, unless the two bodies have the same temperature, an illustration of the fact that equality of action and reaction does not hold between the radiating and receiving bodies alone. They no longer constitute the whole of the momentum system. The ether, or whatever we term the light-bearing medium, is material, and takes its part in the momentum relations of the system.

If the surfaces are partially or totally reflecting, the forces are easily obtained. Thus if one is totally reflecting, it can be shown that the force is only half as great as when it is fully absorbing. But it will be sufficient to confine ourselves to the case of complete absorption, followed by radiation of the absorbed heat equally in all directions from all parts of the surface. More general assumptions do not alter the order of the forces found.

If G is the constant of gravitation $= 6{\cdot}67 \times 10^{-8}$, and if ρ, ρ' are the densities of A and B, the gravitation pull is $G\,\dfrac{16\pi^2 a^3 b^3 \rho\rho'}{9\,d^2}$.

Then on B

$$\frac{\text{Radiation push F}}{\text{Gravitation pull P}} = \frac{9\pi a^2 b^2 R}{16 G U \pi^2 a^3 b^3 \rho\rho'},$$

or

$$\frac{F}{P} = \frac{9R}{16 G U \pi a b \rho\rho'}.$$

If $a = b$; $\rho = \rho'$; $R = 5{\cdot}32 \times 10^{-5}\theta^4$, we have

$$a = \frac{2{\cdot}18\theta^2 \times 10^{-4}}{\rho}\sqrt{\frac{P}{F}}.$$

If we suppose the two bodies to have the temperature of the sun say, 6200° A, and its density, say 0·25, then $F = P$, when

$$a^2 = \frac{4{\cdot}75 \times 6200^4 \times 10^{-8}}{0{\cdot}25^2},$$

then $a = 33{,}500$ centims. or 335 metres.

Of course two globes of this size would soon cool far below the temperature of the sun, even if for an instant they could be raised up to it.

If we suppose $\theta = 300°$ A—the approximate temperature of small bodies at the distance of the earth from the sun—and if we take $\rho = 1$, then $F = P$, when $a = 19{\cdot}62$ centims.

Thus two globes of water—probably nearly full absorbers at 300° A—will at that temperature neither attract nor repel each other if their radii are about 20 centims.

If the density of the spheres is 11, about that so often used for masses in the Cavendish experiment, $F = P$ when

$$a = 1{\cdot}78 \text{ centims.}$$

This does not throw any doubt on the results of Cavendish experiments, for it only holds when the radiators are in an enclosure of very low absolute temperature. In all Cavendish experiments the greatest care is taken to make the attracted body and its enclosure of one uniform temperature.

The really interesting case is that of two small meteorites, in interplanetary space. To judge from the specimens which succeed in penetrating the earth's atmosphere they are very dense. Let us suppose them to have density 5·5—that of the earth—and temperature 300° A, that which they will have at the earth's distance. Then F = P when

$$a = 3 \cdot 4 \text{ centims.}$$

If the radii of the bodies are less than the values found for equality of F and P in the different cases, the net effect is repulsion.

The ratio of F to P is inversely as the square of the radius, so that, as the radii are decreased from the values giving F = P, the radiation repulsion soon becomes enormously greater than the gravitation pull, and the latter may be neglected in comparison. Thus for two drops of water at 300° A in a zero enclosure, with radii 0·001 centim., the pressure is nearly 400,000,000 times the pull.

It is not, however, that the radiation force is great, or even its acceleration. The force becomes exceedingly minute, but the gravitation much more minute.

Thus consider two drops of water at 300° placed in a zero enclosure at a distance $d = 10a$ apart. Our assumption of parallel radiation from one to the other is now only a rough approximation, but the result will be of the right order.

The radiation push is $\pi a^4 R/Ud^2$, and the acceleration is $3aR/4Ud^2 = \frac{1}{10^7} \times \frac{1}{a}$ approximately.

This only becomes considerable when the drops approach molecular dimensions, and long before this they cease to absorb fully the stream of momentum falling on them. Still, even molecules are selective absorbers, and absorb especially each other's radiations. And we may expect that if two gas molecules collide and set each other radiating much more violently than before, they will be practically in an enclosure of much lower temperature than their own, and their mutual radiation may result in very rapid repulsion—repulsion of the order of the fourth power of the temperature reached.

Radiation Pressure between Small Bodies at Different Distances from the Sun.

We have seen above, that if two small spheres of density 5·5 are at the distance of the earth from the sun, their gravitation will be balanced by their radiation pressure when the radius of each is 3·4 centims. Now the balancing radius is proportional to the square of the temperature, that is, inversely proportional to the distance, since the temperature (Part I.) is inversely as the square root of the distance. Thus, at the distance of Mercury, the radii would be about 9 centims.; a million miles from the sun's surface they would be about 200 centims.; out at Neptune they would be about 1 millim.

We see then that the mutual action between small bodies of density that of the earth, will, at different distances, change sign for different sizes of body, ranging from something of the order of 4 metres diameter near the sun to the order of 2 millims. diameter at the distance of Neptune. A ring of small planets, each of radius 3·4 centims., and density 5·5, would move round the sun at the distance of the earth without net mutual attraction or repulsion, and each might be regarded as moving independently of the rest. It appears possible that if Saturn is hot enough, considerations of this kind may apply to his rings.

The repulsion between small colliding bodies, even if not heated by the sun, must lead to some delay in their final aggregation. This is obvious when there are only two small bodies, and their temperature is very considerably raised by the collision. But there is also delay, if instead of a single pair we suppose two swarms to collide. Near the boundary of the colliding region, a body will experience radiation pressure chiefly on one side, and will tend to be driven out of the system. Of course, if the swarms are so dense that a member near the outside cannot see through the rest, this effect will be less. A body in front of another entirely screens its radiation, but the gravitation is not screened. Hence, a body near the boundary of a densely-packed region of collision may be repelled only by the colliding bodies just round it, while it will be attracted by all; or, to put the same idea in another way, a body in a spherical swarm of uniform temperature will only be pulled equally in all directions at the centre of the swarm, but it will be equally repelled in all directions as soon as it is sufficiently deep to be surrounded by its fellows wherever, so to speak, it looks.

Inequality of Action and Reaction between Two Mutually Radiating Bodies.

We have seen that two distant spheres push each other with forces $\pi a^2 b^2 \mathrm{R}/\mathrm{U}\, d^2$ and $\pi a^2 b^2 \mathrm{R}'/\mathrm{U}\, d^2$, and that these, though opposite, are not equal unless $\mathrm{R} = \mathrm{R}'$.

It would be easy to imagine cases in which the forces were not even opposite or in the same directions. At first sight, then, it would appear that we have two bodies acting upon each other with unequal forces, but of course this statement is inexact. The bodies do not act upon each other at all; each sends out a stream of momentum into the medium surrounding it. Some of this momentum is ultimately intercepted by the other, and in its passage the momentum belongs neither to one body nor to the other. If we assume that the momentum is conserved, and of course everything in the methods of this paper depends on that assumption, the action on one of the bodies is equal and opposite to the reaction on the light-bearing medium contiguous to it. There is no failure of the law of action and reaction, but an extension of our idea of matter to include the medium. There should be no difficulty in this extension; indeed, we have made it long ago in endowing the medium with energycarrying properties. Whether the momentum in the medium is in the form of mass m moving with velocity v in the direction of propogation is perhaps open to doubt. We may, perhaps, have different forms of momentum just as we may have different forms of energy, and possibly we ought not to separate the momentum in radiation into the factors m and v, but keep it for the present as one quantity M.

An interesting example of inequality of the radiation forces on two mutually radiating bodies is afforded by two equal spheres, for which, at a given temperature, the radiation push F balances the gravitation pull P. Raise one in temperature so that the push on the other becomes F′. Lower the other so that the push on the first becomes F″, but adjust so that

$$F' + F'' = 2F = 2P,$$

then

$$P - F'' = F' - P.$$

There will then be equal accelerations of the two in the same, *not* in opposite directions, and a chase will begin in the line joining the centres, the hotter chasing the colder. If the two temperatures could be maintained, the velocity would go on increasing; but the increase would not be indefinitely great, inasmuch as a Doppler effect would come into play. Each sphere moving forward would crowd up against the radiation it emitted in front, and open out from the radiation it emitted backwards. This would increase the front and decrease the back pressure, and ultimately the excess of front pressure would balance the accelerating force due to mutual radiation.

Let us examine the effect of motion of a radiating surface on the pressure of its radiation against it.

Application of DOPPLER'S *Principle to the Radiation Pressure against a Moving Surface.*

If a unit area A, fig. 4, is moving with velocity u in any direction AB, making angle ψ with its normal AN, the effect on the energy density in the stream of radiation issuing in any direction AP is two-fold. If the motion is such as to shorten AP, the waves and their energy are crowded up into less space, and if such as to lengthen AP, they are opened out. At the same time, in the one case A is doing work against the radiation pressure and in the other is having work done on it. We shall assume, as in the thermodynamic theory of radiation, that this work adds to or subtracts from the energy of radiation. Both effects (1) the crowding, and (2) the work done, or the reverse of each, combine to alter the energy and therefore the radiation pressure. We have no data by which we can determine whether the

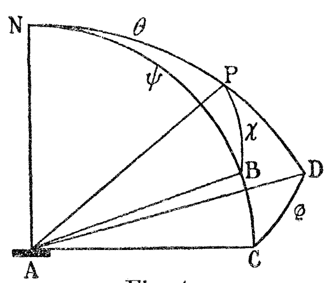

Fig. 4.

motion alters the rate at which the surface is emitting radiation, but it appears worth while to trace consequences on the assumption that the radiation goes on as if the surface were at rest,* but that it is crowded up into less space or spread over more, and that we can superpose on this the energy given out to, or taken from, the stream by the work done by or on the moving surface by the radiation pressure. This work can evidently be calculated to the first order of approximation by supposing the pressure equal to its value when the surface is at rest.

Let us draw from A as centre a sphere of radius U, equal to the velocity of radiation. The energy which, in a system at rest, would be radiated into a cone with A as vertex, length U, and solid angle $d\omega$, in the direction AP making χ with the direction of motion AB, will now be crowded up into a cone of length $\mathrm{U} - u\cos\chi$, since $u\cos\chi$ is the velocity of A in the direction AP. We shall suppose that u/U is very small. Hence the energy density in the cone is increased in the ratio $\mathrm{U} + u\cos\chi : \mathrm{U}$ or by the factor $1 + u\cos\chi/\mathrm{U}$.

Considering now the effect of the work done, the force on A due to the stream in $d\omega$ is $\mathrm{N}\cos\theta\, d\omega/\mathrm{U}$, and the work done in one second is $(\mathrm{N}\cos\theta\, d\omega/\mathrm{U}) \times u\cos\chi$.

When A is at rest the energy in this cone is

$$\mathrm{N}\cos\theta\, d\omega.$$

When A is moving it is increased to

$$\mathrm{N}\cos\theta\, d\omega + \frac{\mathrm{N}\cos\theta\, d\omega}{\mathrm{U}}\, u\cos\chi,$$

that is

$$\mathrm{N}\cos\theta\, d\omega\left(1 + \frac{u\cos\chi}{\mathrm{U}}\right).$$

Thus the effect of the work done is equal to that of the crowding and the energy density on the whole is increased in the ratio

$$1 + \frac{2u\cos\chi}{\mathrm{U}} : 1.$$

The pressure is increased in the ratio of the energy density. Then the force on A due to the radiation through $d\omega$ is increased from

$$\frac{\mathrm{N}\cos\theta\, d\omega}{\mathrm{U}} \quad \text{to} \quad \frac{\mathrm{N}\cos\theta\, d\omega}{\mathrm{U}}\left(1 + \frac{2u\cos\chi}{\mathrm{U}}\right).$$

* *Added August* 20, 1903.—Since the above was written Professor LARMOR has pointed out to me that the results obtained in the text from this assumption, along with the hypothesis of crowding of the radiation and its increase by an amount equivalent to the work of the radiation pressure, can be justified by an argument based on the following considerations. A perfect reflector moving with uniform speed in an enclosure, itself also moving at that speed, and so in a steady state, must send back as much radiation of every kind as a full radiator in its place. Now the electrodynamics of perfect reflexion are known; hence the effect of motion of a full radiator on the amount of its radiation can be determined. The result is equivalent to the statement that the amplitudes of the excursions of the optical vibrators are the same at the same temperature whether the source to which they belong is moving or not.

If we resolve this along the normal to the surface A and integrate over the hemisphere we obtain the total normal pressure. As we only want to know the change in pressure P we may neglect the first term which gives the pressure on A at rest, and we have

$$P = \int \frac{N\cos^2\theta}{U} \cdot \frac{2u\cos\chi}{U}\,d\omega.$$

If ϕ is the angle between the normal planes through B and P we have

$$\cos\chi = \cos\theta\cos\phi + \sin\theta\sin\phi\cos\phi.$$

Putting $d\omega = \sin\theta\,d\theta\,d\phi$,

$$P = \int_0^{\frac{\pi}{2}}\int_0^{2\pi} \frac{2uN}{U^2}\cos^2\theta\sin\theta\,(\cos\theta\cos\psi + 2\theta\sin\psi\cos\phi)\,d\theta\,d\phi$$

$$= \frac{\pi N u\cos\psi}{U^2} \qquad = \frac{Ru\cos\psi}{U^2}.$$

The change in the tangential stress is evidently in the direction AC, that of the component of u in the plane of A.

We may therefore resolve each element of tangential stress in the direction AC. Omitting the first term again, since in this case it disappears on integration, the element due to $d\omega$ in the direction AP will contribute

$$\frac{N\cos\theta\sin\theta\cos\phi}{U} \cdot \frac{2u\cos\chi}{U}\,d\omega,$$

and integrating over the hemisphere we have

$$T = \int_0^{\frac{\pi}{2}}\int_0^{2\pi} \frac{2uN}{U^2}\cos\theta\sin^2\theta\cos\phi\,(\cos\theta\cos\psi + \sin\theta\sin\psi\cos\phi)\,d\theta\,d\phi$$

$$= \frac{\pi N u\sin\psi}{2U^2} \qquad = \frac{Ru\sin\psi}{2U^2}.$$

Force on a Sphere moving with Velocity "u" in a Given Direction.

If a sphere, radius a, is moving with velocity u, we may from symmetry resolve the forces on each element in the direction of motion. The resolutes will be $P\cos\psi$ and $T\sin\psi$. Evidently it is sufficient to integrate over the front hemisphere and then double the result. We have the

$$\text{Retarding Force} = 2\int_0^{\frac{\pi}{2}}\left(\frac{Ru\cos^2\psi}{U^2} + \frac{Ru\sin^2\psi}{2U^2}\right)2\pi a^2\sin\psi\,d\psi$$

$$= \tfrac{8}{3}\frac{Ru}{U^2}\cdot\pi a^2.$$

It is noteworthy that one half of this is due to the normal, the other half to the tangential stresses.

If the sphere has density ρ the acceleration is obtained by dividing by $\frac{4}{3}\pi a^3\rho$, then

$$du/dt = -2Ru/U^2\rho a.$$

Effect on Rotation.

If the sphere radius a is rotating with angular velocity ω, then any element of the surface λ from the equator is moving with linear velocity at $\omega \cos \lambda$ in its own plane. This does not affect the normal pressure, but it introduces a tangential stress opposing the motion

$$Ru/2U^2 = Ra\omega \cos \lambda/2U^2.$$

Taking moments round the axes and integrating over the sphere, we obtain a couple

$$\tfrac{4}{3}\pi a^3 . \tfrac{2}{5} a^2 \frac{d\omega}{dt} = \frac{Ra\omega}{2U^2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2\pi a^3 \cos^3 \lambda \, d\lambda,$$

whence

$$d\omega/dt = -\tfrac{5}{2} R\omega/2U^2\rho a.$$

The rate of diminution of ω is therefore of the same order as that of u.

To obtain an idea of the magnitude of the retardation of a moving sphere, let us suppose that one is moving through a stationary medium. Let its radius be $a = 1$ centim., its density $\rho = 5 \cdot 5$, its temperature 300° A.

Then

$$\frac{1}{u}\frac{du}{dt} = -\frac{2 \times 5 \cdot 32 \times 10^{-5} \times 300^4}{9 \times 10^{20} \times 5 \cdot 5}$$

$$= 1 \cdot 75 \times 10^{-16}.$$

This will begin to affect the velocity by the order of 1 in 10,000 in, say, 10^{12} seconds, or taking the year as $3 \cdot 15 \times 10^7$ seconds, in about 30,000 years.

The effect is inversely as the radius, so that a dust particle 0·001 centim. radius will be equally affected in 30 years.

The effect is as the fourth power of the temperature, so that with rising temperature it becomes rapidly more serious.

Equation to the Orbit of a Small Spherical Absorbing Particle Moving in a Stationary Medium Round the Sun.

It is evident from the above result, that the effect of motion on radiation pressure may be very considerable in the case of a small absorbing particle moving round the sun.

We shall take the particle as spherical, of radius a and distance r from the sun. We shall suppose the radius so small that the particle is of one temperature throughout, the temperature due to the solar radiation which it receives, but that it is still so large as to be attracted much more than it is repelled by the sun. Both attraction and repulsion are inversely as the square of the distance, so that we shall have a central force which we may put as producing acceleration A/r^2, where A is constant.

We know that at the distance of the earth, putting $r = b$, $A/b^2 = 0{\cdot}59$ centim./sec.^{2}, say $0{\cdot}6$ centim./sec.^{2} Then $A = 0{\cdot}6b^2$. The force acting against the motion produces retardation $-2Ru/U^2\rho a$.

If S is the solar constant at the distance b, its value at distance r is

$$Sb^2/r^2.$$

Putting

$$4\pi a^2 R = \pi a^2 \, Sb^2/r^2$$

$$R = (S/4)\,(b^2/r^2),$$

then the acceleration in the line of motion is

$$-\frac{Sb^2}{2U^2\rho a}\cdot\frac{u}{r^2} = -\frac{T\dot{s}}{r^2},$$

where $T = Sb^2/2U^2\rho a$, and $\dot{s}$ is now written for the velocity u.

The accelerations along and perpendicular to the radius vector give the equations

$$\ddot{r} - r\dot{\theta}^2 = -\frac{A}{r^2} - \frac{T\dot{s}}{r^2}\frac{dr}{ds} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1),$$

$$\frac{1}{r}\frac{d}{dt}(r^2\dot{\theta}) = -\frac{T\dot{s}}{r^2}\frac{rd\theta}{ds} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

From (2) we get

$$\frac{d}{dt}(r^2\dot{\theta}) = -\frac{Td\theta}{dt},$$

whence

$$r^2\dot{\theta} = C - T\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3),$$

where C is the constant of integration.

If θ is 0 when $t = 0$, then C is the initial value of $r^2\dot{\theta}$. Further, as θ increases $r^2\dot{\theta}$ decreases and is 0 when $\theta = C/T$. This gives a limit to the angle described.

Equation (1) may be written

$$\ddot{r} - r\dot{\theta}^2 = -A/r^2 - T\dot{r}/r^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4).$$

Putting u for r^{-1}

$$\dot{r} = \frac{dr}{d\theta}\dot{\theta} = -\frac{1}{u^2}\frac{du}{d\theta}\dot{\theta} = -(C - T\theta)\frac{du}{d\theta} \text{ from (3)},$$

$$\ddot{r} = T\dot{\theta}\frac{du}{d\theta} - (C - T\theta)\frac{d^2u}{d\theta^2}\dot{\theta}$$

$$= T(C - T\theta)\,u^2\frac{du}{d\theta} - (C - T\theta)^2 u^2 \frac{d^2u}{d\theta^2} \text{ from (3)}$$

Substituting in (4)

$$\frac{d^2u}{d\theta^2} + u = \frac{A}{(C - T\theta)^2}.$$

This can probably only be integrated by approximation. We can see the effect on the motion at the beginning by putting

$$\frac{d^2u}{d\theta^2} + u = \frac{A}{C^2}\left(1 + \frac{2T}{C}\,\theta\right),$$

since T/C is small if we begin at the distance of the earth and with a particle having the velocity of the earth.

An integral of this is

$$u = \frac{A}{C^2}\left(1 + \frac{2T}{C}\,\theta\right).$$

The complementary function will be periodic and may be omitted. To the order of approximation adopted

$$r = \frac{C^2}{A}\left(1 - \frac{2T}{C}\,\theta\right) \qquad \text{and} \qquad \dot{r} = -\,\frac{2CT}{A}\,\dot{\theta}.$$

Then initially

$$\dot{r}/r = -\,(2T/C)\,\dot{\theta}.$$

In applying these results, we may note that $T = Sb^2/2U^2\rho a$ is constant for all distances, and that b, the earth's distance, is 493 U. Inserting the value of the solar constant, $0{\cdot}175 \times 10^7$, and taking $\rho = 5{\cdot}5$, we get

$$T = 3{\cdot}9 \times 10^{10} \,.\, a^{-1}.$$

C will depend on the initial conditions. Assuming that the body considered is initially moving in a circle, then, at the beginning

$$r\dot{\theta}^2 = \frac{A}{r^2} \quad \text{or} \quad \dot{\theta} = \sqrt{\frac{A}{r^3}} = \sqrt{\frac{0{\cdot}6b^2}{r^3}},$$

since at $r = b$ the acceleration to the centre is $0{\cdot}6$.

Then

$$C = r^2\dot{\theta} = \sqrt{0{\cdot}6b^2r}.$$

Substituting these values in $\dot{r}/r$ we have

$$\frac{\dot{r}}{r} = -\,\frac{7{\cdot}8 \times 10^{10}}{r^2 a}.$$

This gives only the initial value of $\frac{\dot{r}}{r}$ and cannot be taken to hold for a time which will make $T^2\theta^2/C^2$ appreciable. But by (3) we see that $r = 0$ if $\theta = C/T$, so that

$C/2\pi T$ is a superior limit to the number of revolutions, even if we suppose the way clear right up to the centre.

Putting the numerical values we get

$$C/2\pi T = 61 r^{\frac{1}{2}} a.$$

Suppose, for example, that $r = b = 493 \times 3 \times 10^{10}$; $a = 1$, then

$$\dot{r}/r = -3{\cdot}5 \times 10^{16}.$$

If we multiply by $3{\cdot}15 \times 10^7$, the seconds in a year, $(\dot{r}/r) \times 3{\cdot}15 \times 10^7 = 1{\cdot}1 \times 10^{-8}$.

This implies that a sphere 1 centim. radius and density 5·5, starting with the velocity of the earth, and at its distance from the sun, will move inwards $\frac{1}{10,000}$ of its distance in about 10,000 years. It cannot in all make so many as $61 \times b^{\frac{1}{2}} = 2{\cdot}35 \times 10^8$ revolutions.

If we put $a = 0{\cdot}001$ centim., since the effects are inversely as a, then its distance will decrease by about 1 in 10,000 in 10 years, and it cannot make in all so many as $2{\cdot}35 \times 10^5$ revolutions.

If instead of starting from the distance of the earth, the particle starts from, say, 0·1 the distance, the effect in the radius is 100 times as great and the number of revolutions is $\sqrt{10}$ times less. Then with radius 1 centim. the distance decreases by $\frac{1}{10,000}$ in 100 years, and there are not so many as 80,000 revolutions, while with radius 0·001 centim. the distance decreases by $\frac{1}{10,000}$ in 0·1 year, and there are not so many as 80 revolutions.

Small particles, therefore, even of the order of 1 centim. radius, would be drawn into the sun, even from the distance of the earth, in times not large compared with geological times, and dust particles if large enough to absorb solar radiation would be swept in in a time almost comparable with historical times. Near the sun the effects are vastly greater. The application to meteoric dust in the system is obvious.

There should be a similar effect with dust and small particles circulating round the earth. If, for example, any of the Krakatoa dust was blown out so far beyond the appreciable atmosphere, and was given such motion that the particles became satellites to the earth, at no long time the dust will return. A ring of dust particles moving round a planet and receiving heat either from the sun or from the planet will tend to draw in to the planet.

[*Note added October* 31.—Since the foregoing paper was printed I have re-examined the theory of the pressure on a fully radiating surface when in motion, and have come to the conclusion that the change in pressure due to the motion is only half as great as that obtained on p. 545. In that investigation the pressure was assumed to be equal to the energy density, whether the surface was at rest or in motion, whereas it appears, if the following mode of treatment is correct, that the pressure on a radiating surface moving forward is only $1 - \frac{u}{U}$ of the energy density of the radiation emitted.

Let us suppose that a surface A, a full radiator, is moving with velocity u towards a full absorber B, which, with the surroundings, we will suppose at 0° A. Consider for simplicity a parallel pencil issuing normal from A with velocity U towards B. Let the energy density in the stream from A be E when A is at rest, and E′ when it is moving. Let the pressure on A be $p = E$ when it is at rest, and p' when it is moving. When moving, A is emitting a stream of momentum p' per second and this momentum ultimately falls on B. Let A start radiating and moving at the same instant; let it move a distance d towards B, and then let it stop radiating and moving. It emits momentum p' per second for a time d/u and therefore emits total momentum $p'd/u$. Since B is at rest, the pressure on it, the momentum which it receives per second, is E′. But since A is following up the stream sent out, B does not receive through a period as long as d/u, but for a time less by d/U. If we assume that the total momentum received by B is equal to the total sent out by A, we have

$$p'd/u = E'(d/u - d/U),$$

or

$$p' = E'(1 - u/U).$$

To find E′ in terms of E we must make some assumption as to the effect of the motion on the radiation emitted. In the paper I have assumed that the emitting surface converts the same amount of its internal energy per second into radiant energy as when it is at rest, but that $p'u$ of the energy of motion of the radiating mass is also converted into radiant energy. Since the radiation emitted in one second is contained in length $U - u$, we have

$$E'(U - u) = EU + p'u = EU + E'\left(\frac{U - u}{U}\right)u,$$

whence

$$E' = E\frac{U^2}{(U - u)^2} = E(1 + 2u/U).$$

The same result is obtained if we assume that the amplitude of the emitted waves is the same whether the surface is moving or not, and that the energy density is inversely as the square of the wave-length for given amplitude.

We have, therefore, if the above application of the equality of action and reaction is justified,

$$p' = E'\left(1 - \frac{u}{U}\right) = E\frac{U}{U - u} = p\left(1 + \frac{u}{U}\right).$$

In a similar way we can find the effect of motion of an absorber on the pressure against it due to the incident radiation.

Let a stream of energy density E be incident on a fully absorbing surface moving towards the source with velocity u. Let the surface be at 0°A, so as to obtain the effect of the incident radiation only. When the surface is at rest, we may regard the stream as bringing up momentum E per second, or as containing momentum of density E/U brought up with velocity U to it. If the surface is moving towards the source, it takes up in one second the momentum in length $\mathrm{U}+u$, or receives $\frac{\mathrm{E}}{\mathrm{U}}(\mathrm{U}+u)$, and the pressure on it is $p' = \mathrm{E}\left(1+\frac{u}{\mathrm{U}}\right) = p\left(1+\frac{u}{\mathrm{U}}\right)$.

It is easy to show that when a perfect reflector is moving, the pressure upon it is altered from p to $p\left(1+\frac{2u}{\mathrm{U}}\right)$.

In the paper, the case of a full radiator in an enclosure at zero has alone been considered, so that the correcting factor is $1+\frac{u}{\mathrm{U}}$ or $1+\frac{u\cos\chi}{\mathrm{U}}$ when the motion is at χ to the line of radiation. Hence the forces obtained in the paper when the factor was $1+\frac{2u}{\mathrm{U}}$ are all double those obtained with the factor now given. The process of drawing in small particles to the sun is correspondingly lengthened out.

It is, perhaps, worth noting that the motion of a body round the sun produces a small aberration effect. If the body is a sphere, the sunlight does not fall on the hemisphere directly under the sun, but on one turned round through an angle u/U. The pressure of the radiation, though still straight from the sun, does not act through the centre but through a point $\frac{u^2}{2\mathrm{U}^2}\times$ radius of sphere in front of the centre. Thus, in the case of the earth, it will tend to stop the rotation. But the effect is so minute that if present conditions as to distance and radiation were maintained, it would take something of the order of 10^{19} years to stop the whole of the rotation.]