Keplerian Elements for Approximate Positions of the Major Planets

E M Standish  
Solar System Dynamics Group  
JPL / Caltech

Lower accuracy formulae for planetary positions have a number of important applications when one doesn’t need the full accuracy of an integrated ephemeris. They are often used in observation scheduling, telescope pointing, and prediction of certain phenomena as well as in the planning and design of spacecraft missions.

Approximate positions of the nine major planets may be found by using Keplerian formulae with their associated elements and rates. Such elements are not intended to represent any sort of mean; they are simply the result of being adjusted for a best fit. As such, it must be noted that the elements are not valid outside the given time-interval over which they were fit.

The elements are given below in Table 1 or in Tables 2a and 2b, depending upon the time-interval over which they were fit and within which they are to be used.

Formulæ for using the Keplerian elements

Keplerian elements given in the tables below are

\[ a_0, \dot{a} \quad : \text{semi-major axis [au, au/century]} \]
\[ e_0, \dot{e} \quad : \text{eccentricity [ , /century]} \]
\[ I_0, \dot{I} \quad : \text{inclination [degrees, degrees/century]} \]
\[ L_0, \dot{L} \quad : \text{mean longitude [degrees, degrees/century]} \]
\[ \varpi_0, \dot{\varpi} \quad : \text{longitude of perihelion [degrees, degrees/century] (\varpi = \omega + \Omega)} \]
\[ \Omega_0, \dot{\Omega} \quad : \text{longitude of the ascending node [degrees, degrees/century]} \]

In order to obtain the coordinates of one of the planets at a given Julian Ephemeris Date, \( T_{eph} \),

1. Compute the value of each of that planet’s six elements: \( a = a_0 + \dot{a}T \), etc., where \( T \), the number of centuries past J2000.0, is \( T = (T_{eph} - 2451545.0)/365.25 \).

2. Compute the argument of perihelion, \( \varpi \), and the mean anomaly, \( M \) :

\[ \varpi = \varpi - \Omega; \quad M = L - \varpi + \dot{\varpi}T^2 + c \cos(fT) + s \sin(fT) \quad (8-30) \]

where the last three terms must be added to \( M \) for Jupiter through Pluto when using the formulæ for 3000 BC to 3000 AD.

3. Modulus the mean anomaly so that \(-180^\circ \leq M \leq +180^\circ \) and then obtain the eccentric anomaly, \( E \), from the solution of Kepler’s equation (see below):

\[ M = E - e^* \sin E, \quad (8-31) \]

where \( e^* = 180/\pi e = 57.29578 e \).

4. Compute the planet’s heliocentric coordinates in its orbital plane, \( r' \), with the \( x' \)-axis aligned from the focus to the perihelion:
\[ x' = a(\cos E - e) \quad y' = a\sqrt{1 - e^2} \sin E \quad z' = 0. \] (8 - 32)

5. Compute the coordinates, \( r_{\text{ecl}} \), in the J2000 ecliptic plane, with the \( x \)-axis aligned toward the equinox:

\[
r_{\text{ecl}} = M r' \equiv \mathcal{R}_z(-\Omega)\mathcal{R}_x(-I)\mathcal{R}_z(-\omega)r'
\]
so that
\[
x_{\text{ecl}} = (\cos \omega \cos \Omega - \sin \omega \sin \Omega \cos I) \ x' + (-\sin \omega \cos \Omega - \cos \omega \sin \Omega \cos I) \ y' \]
\[
y_{\text{ecl}} = (\cos \omega \sin \Omega + \sin \omega \cos \Omega \cos I) \ x' + (-\sin \omega \sin \Omega + \cos \omega \cos \Omega \cos I) \ y' \]
\[
z_{\text{ecl}} = (\sin \omega \sin I) \ x' + (\cos \omega \sin I) \ y' \] (8 - 34)

6. If desired, obtain the equatorial coordinates in the “ICRF”, or “J2000 frame”, \( r_{\text{eq}} \):

\[
x_{\text{eq}} = x_{\text{ecl}}
\]
\[
y_{\text{eq}} = + \cos \varepsilon \ y_{\text{ecl}} - \sin \varepsilon \ z_{\text{ecl}}
\]
\[
z_{\text{eq}} = + \sin \varepsilon \ y_{\text{ecl}} + \cos \varepsilon \ z_{\text{ecl}}
\] (8 - 35)

where the obliquity at J2000 is \( \varepsilon = 23^\circ 27' 28'' \).

**Solution of Kepler’s Equation, \( M = E - e^* \sin E \)**

Given the mean anomaly, \( M \), and the eccentricity, \( e^* \), both in degrees, start with

\[
E_0 = M + e^* \sin M
\] (8-36)

and iterate the following three equations, with \( n = 0, 1, 2, \ldots \), until \( |\Delta E| \leq \text{tol} \) (noting that \( e^* \) is in degrees; \( e \) is in radians):

\[
\Delta M = M - (E_n - e^* \sin E_n) \quad \Delta E = \Delta M/(1 - e \cos E_n) \quad E_{n+1} = E_n + \Delta E.
\] (8-37)

For the approximate formulae in this present context, \( \text{tol} = 10^{-8} \text{degrees} \) is sufficient.

**Table 1**

Keplerian elements and their rates, with respect to the mean ecliptic and equinox of J2000, valid for the time-interval 1800 AD - 2050 AD.

<table>
<thead>
<tr>
<th></th>
<th>( a ) [au, au/cty]</th>
<th>( e )</th>
<th>( I ) [deg, deg/cty]</th>
<th>( L ) [deg, deg/cty]</th>
<th>( \omega ) [deg, deg/cty]</th>
<th>( \Omega ) [deg, deg/cty]</th>
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<td>0.20563593</td>
<td>7.00497902</td>
<td>252.25032350</td>
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<td>-0.00001531</td>
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2
Table 2a

Keplerian elements and their rates, with respect to the mean ecliptic and equinox of J2000, valid for the time-interval 3000 BC – 3000 AD. **NOTE**: the computation of $M$ for Jupiter through Pluto **must** be augmented by the additional terms described above and given in Table 2b.

<table>
<thead>
<tr>
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Table 2b

Additional terms which must be added to the computation of $M$ for Jupiter through Pluto, 3000 BC to 3000 AD, as described above.

<table>
<thead>
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<th>$b$</th>
<th>$c$</th>
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